

## U-turn Waiting Time Estimation at Midblock Median Opening

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**Abstract:** The u-turn traffic at midblock median opening has to wait for the acceptable headway of conflicting through traffic stream. This research proposed the waiting time estimation by spreadsheet simulation. The spreadsheet software generated random headway size according to the known statistic distribution. The randomly generated headways were compared with the critical headway, which is assumed to follow the log-normal distribution, to determine whether the u-turn vehicle accepts the headway. The data was collected at a midblock median opening in order to evaluate the estimation result. The results showed that the proposed method can estimate the waiting time well when the through traffic volume is not large. When the through traffic volume is large, the collected waiting time is much lower than its estimated value. The result implied that the u-turn drivers became impatient while waiting for a long time and conducted the unsafe forcing u-turn maneuver.

*Keywords:* U-turn, Waiting Time, Unsignalized Intersection, Simulation, Gap Acceptance

### 1. INTRODUCTION

There are a lot of u-turn facilities at the midblock median opening on urban arterials in developing countries. When the u-turn vehicles arrive at the u-turn junction, they wait for an acceptable gap in the main through traffic stream and complete the u-turn maneuver. The behavior of u-turn movement is similar to the movement of the minor street traffic at the unsignalized intersections, which is the “gap acceptance process”. The u-turn movement has a better sight distance; however, the u-turn vehicles’ maneuvering needs more time and larger space. As the u-turn movement is a non-priority traffic hierarchy, the waiting of the drivers affect their decisions. The longer waiting time the drivers face, the higher chance the drivers take a risk and accept a short gap.

Typically, the delay represents the service quality of the unsignalized intersection. The delay time comprises two parts, which are queuing time and waiting time. The queuing time is the excess time that spending in line of queue. On the other hand, the waiting time is the time that spending at the front position of the queue or at the stop line. For u-turn traffic, the waiting time significantly affects the u-turn decision while the queuing time does not. The vehicle at the first position of the queue gets pressure from other following vehicles, especially during rush hours. Therefore, the waiting time could be a key factor characterizing the traffic operation at the u-turn junction. U-turn drivers may not wait and conduct a forcing movement after waiting for a long time. Such a risky operation needs some kinds of control. Normally, the traffic police will present at the congested u-turn junction to direct the u-turn traffic. Knowing the waiting time relationship will be useful for safer u-turn management.

Based on the gap acceptance process, the amount of waiting time theoretically relates to the traffic characteristics (volume, headway distribution, etc.) of the conflicting through traffic. However, the waiting time might be distorted by the unsafe forcing u-turn when the driver is impatient. Considering this fact, the empirical approach by field data collection can reflect the results of undesirable driver's behaviors, both cautious and aggressive.

Currently, the authorized formula to calculate the theoretical waiting time is not available. This research applied the tailor-made spreadsheet simulation technique to estimate the waiting time. A number of random sets of time-series headway sizes were generated according to the known distribution functions, representing the headway of the through traffic stream in the field. The waiting time of a driver can be determined by accumulating all the rejected headway sizes. This requires a value of critical headway according to the field data collection. This proposed method is a simplified simulation, without using any commercial traffic simulation software packages, which are costly and complex to use. The method can utilize the common spreadsheet software, i.e., Microsoft Excel.

The objective of this research can be listed as follows:

- propose a method to estimate the theoretical u-turn waiting time,
- develop the waiting time model as a function of conflicting traffic volume,
- analyze the data to get the field u-turn waiting time, and
- investigate the effect of driver behaviors on waiting time aspect.

## 2. LITERATURE REVIEW

The related past research works about the waiting time estimation concentrate on the two-way stop-controlled (TWSC) intersections. Kyte *et al.* (1991) studied the delay characteristics of TWSC intersections in both macroscopic and microscopic analysis. They found that the waiting time is affected by the traffic flow rate on the conflicting approaches. The linear relationship has been developed by the regression analysis. The waiting time increases as the conflicting flow rate increases. Madanat *et al.* (1994) developed the waiting time function by probability theory. The expected waiting time at the stop line is the product of the average size of rejected gaps and the expected number of rejected gaps. The process of rejecting sequential gaps is expressed as a geometric distribution. The gap size is assumed to be negative exponential distributed in their study. Al-Omari and Benekohal (1999) developed the linear waiting time models for unsaturated TWSC intersections by empirical approach. The separate models are also developed for different turning movements; right, left, and through. The statistical test unveils that there is no significant differences between the three models.

Chandra *et al.* (2009) analyzed the waiting time at uncontrolled intersections in mixed traffic conditions by microscopic approach. The microscopic analysis considers each individual subject vehicle. The conflicting flow rate as seen by the particular subject vehicle is the number of observed conflicting vehicles divided by the observation time. Some advantages of this method are that the data is not lost by aggregation and the data points are increased. It also reflects the real conflicting flow rate the subject vehicle faces when waiting for an acceptable gap. The function of waiting time is in the exponential form.

Some past studies utilize the waiting time to estimate the critical gap. Polus *et al.* (2003) developed a function of waiting time to estimate the critical gap. After knowing the critical gap, they also proposed the exponential relationship between circulating flow and entry capacity at roundabout. Polus *et al.* (2005) also evaluated the effect of waiting time on critical gaps at roundabout by a binary logit model. Traditionally, the critical gap is estimated by the maximum likelihood method (Tian *et al.*, 1999). The method assumes that (1) the critical gaps

are log-normal distributed and (2) the driver behavior is both homogeneous and consistent. Recently, Wu (2012) proposed a new method to estimate the critical gap, using equilibrium of probabilities for rejected and accepted gaps. The proposed method does not require presumptions regarding the distribution function of critical gaps and driver behavior.

The previous study on u-turn movement shows that the longer time the driver waits at the stop line, the smaller gap the driver accepts. The waiting time of more than 30 seconds will frustrate the drivers to accept the significant smaller gap, which may lead to traffic safety problem (Jenjiwattanakul and Sano, 2011). Another study investigates the factors affecting the u-turn decision. The significant factors include gap size, conflicting speed, and waiting time. The queuing time does not significantly affect the u-turn decision (Jenjiwattanakul and Sano, 2012). Because the waiting time significantly affects the decision of u-turn's drivers, the current study developed the function to estimate the waiting time and set the warrant to control the u-turn movement for safety purpose.

### 3. METHODOLOGY

#### 3.1 Waiting Time Estimation

The proposed method considers the arrival process of the conflicting traffic stream in the chronological order. When conflicting traffic vehicles arrive, they create gaps. The latest HCM has replaced the term “gap” with “headway” (TRB, 2010). This might represent the practical data collection process: record headway rather than gap. The waiting time is the summation of the rejected headway sizes before accepting the large enough headway. Therefore, the determination of the waiting time requires the chronological order of conflicting vehicle headways. The critical headway information is also required to determine which headways are acceptable. The example of the headway size of conflicting traffic versus time of the day is shown in Figure 1.

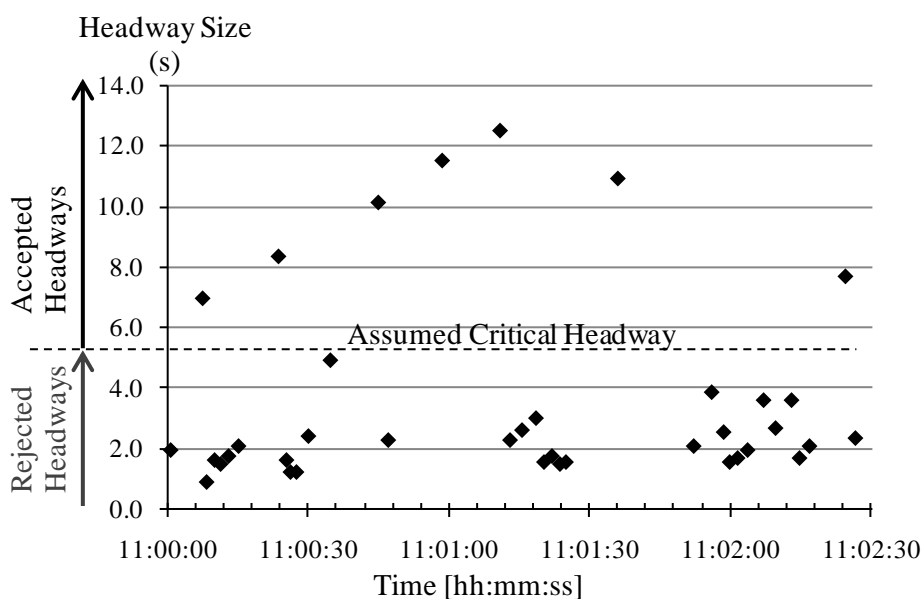


Figure 1. Decision based on chronological order headway events

This research utilized the spreadsheet software (i.e. Microsoft Excel) to generate the random numbers, representing the conflicting vehicle headways. The random numbers are

generated based on the premise probability distributions. The waiting time for each u-turn can be estimated by the summation of the rejected headways. The waiting time for the entire hour can be calculated by the arithmetic mean. The follow-up u-turn vehicles do not wait at the stop line and are excluded from the waiting time calculation.

Based on above, the estimation process required headway distribution function of the through traffic (in order to generate headway), and critical headway characteristics of u-turn traffic (in order to decide whether to accept or reject headway). Those information could be determined from the collected field data. The simulations vary the conflicting traffic volume from 200 to 3,200 vehicles per hour (vph) with the increment of 200 vph. The simulation assumed that u-turn traffic was in a queued condition and was waiting for u-turn all the time; therefore, all rejected headways are considered. The estimation also assumed no follow-up u-turn. The following u-turn vehicle will wait for the next acceptable headway to make u-turn. In addition, the simulation considered the randomness of the critical headway, by assuming log-normal distribution. The accepted headway size varies from driver to driver. Some cautious drivers reject the mean critical headway, while some aggressive drivers accept smaller headway.

The required number of random seeds for each conflicting traffic volume fulfilled the following criteria: (1) a minimum of 15 random seeds and increase in the increment of 5 random seeds; (2) the percentage error from mean not larger than 5%; and (3) the value of error not larger than 1 second. The calculation of prediction error utilized the level of significance of 0.05. For each random seed of conflicting traffic headway, the u-turn critical headway was randomized for 30 seeds.

### 3.2 Critical Headway Determination

The critical headway was determined by the maximum likelihood method (Tian *et al.*, 1999). It assumes that a driver's critical headway is between his largest rejected headway and his accepted headway. The method also assumes a log-normal distribution for the critical headways. The log-likelihood of a sample of  $n$  drivers having an accepted headway and a largest rejected headway of  $(a_i, r_i)$  is given in Eq. (1). After maximizing the log-likelihood function, the mean critical headway and its variance can be calculated from the mean and variance of the distribution of the logarithms of the individual driver's critical headways, as shown in Eq. (2) and Eq. (3), respectively.

$$L = \sum_{i=1}^n \ln[F(y_i) - F(x_i)] \tag{1}$$

where,

- $L$  : logarithm of the likelihood function,
- $y_i$  : logarithm of the accepted headway of the  $i$ th driver =  $\ln(a_i)$ ,
- $x_i$  : logarithm of the largest rejected headway of the  $i$ th driver =  $\ln(r_i)$ , and
- $F()$  : cumulative distribution function of the normal distribution.

$$t_c = e^{\mu+0.5\sigma^2} \tag{2}$$

$$s^2 = t_c^2(e^{\sigma^2} - 1) \tag{3}$$

where,

- $t_c$  : mean critical headway,

- $s^2$  : variance of the critical headway,
- $\mu$  : mean of the distribution of the logarithms of the individual driver's critical headways, and
- $\sigma^2$  : variance of the distribution of the logarithms of the individual driver's critical headways.

### 3.3 Data Extraction: Conflicting Flow Rate and Waiting Time

The conflicting flow rate, extracted from field data, was calculated using microscopic analysis as described in Kyte *et al.* (1999) or Chandra *et al.* (2009). The microscopic analysis considers each u-turn vehicle individually. The amount of conflicting through vehicles that each u-turn vehicle faces during waiting until moving are converted into hourly flow rate, including the last conflicting vehicle that form the acceptable headway, as illustrated in Eq. (4). The waiting time of u-turn vehicle can be determined by the difference between departure time and arrival time, as shown in Eq. (5).

$$q_c = \frac{n}{(t_n - t_0)} \times 3600 \tag{4}$$

$$t_s = t_d - t_0 \tag{5}$$

where,

- $q_c$  : hourly conflicting flow rate,
- $n$  : number of conflicting vehicles, including the last conflicting vehicle that comes just after u-turn movement,
- $t_0$  : time of arrival of the particular u-turn vehicle,
- $t_d$  : time of departure of the particular u-turn vehicle,
- $t_n$  : time of arrival of the last conflicting through vehicle, and
- $t_s$  : waiting time of the particular u-turn vehicle,

## 4. DATA COLLECTION

The traffic data were collected at a midblock median opening on Phetkasem Road, a six-lane urban arterial in western Bangkok, Thailand. Figure 2 illustrates the site location and physical road configuration. The site is located between junctions of Ratchapruk Road and Kanchanaphisek Road (Motorway No. 9).

The road, at the u-turn junction, has three through lanes in each direction with an exclusive u-turn lane on both directions. Most u-turn vehicles encroach to the middle lane in order to complete the u-turn maneuver. The data collection was conducted during morning peak period (07:00-09:00), off-peak period (11:00-13:00), and afternoon peak period (16:00-18:00) on two days. A digital video camera was set up on the pedestrian bridge to record the traffic movements. The recorded video files were reviewed in the laboratory to extract the required data for further analysis.

The data extraction was based on the timestamp of all traffic movement events. The conflicting flow rate and u-turn waiting time were determined by the calculation from the recorded time, as described in the previous section. The headway was the time difference of the passing of two consecutive vehicles on all conflicting lanes. In summary, the sets of data that required for this research include:

- the headway of conflicting through traffic in order to determine the distribution,

- a pair of largest rejected headway and accepted headway for each u-turn vehicle in order to estimate the mean and variance of critical headway, and
- a pair of conflicting flow rate and waiting time for each u-turn vehicle in order to find their relationship.

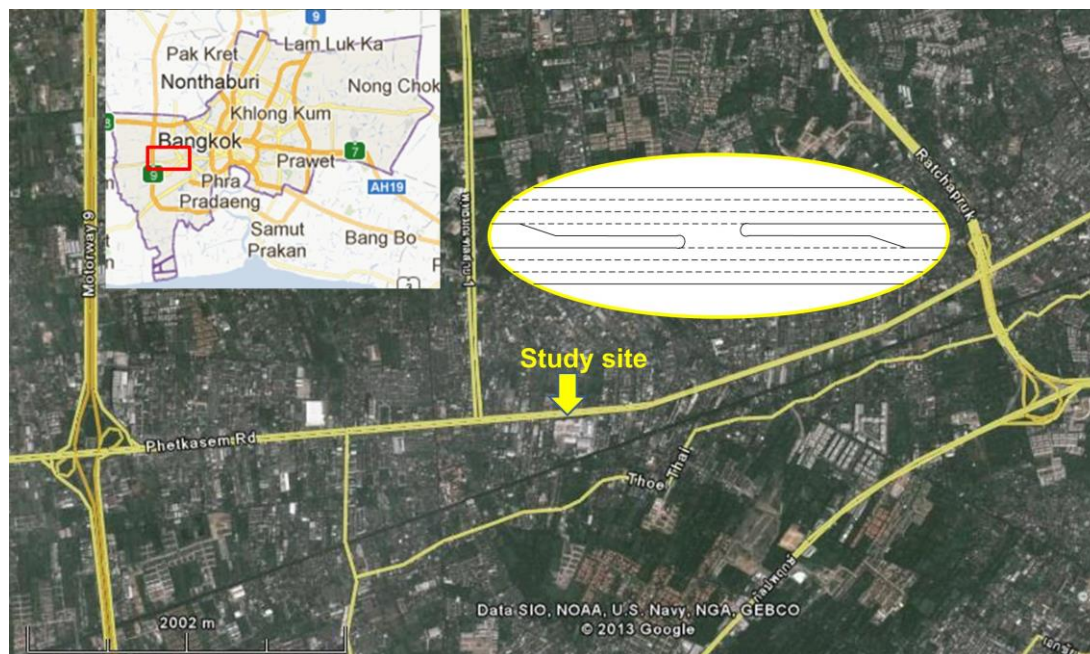


Figure 2. Study site location and road configuration

## 5. RESULTS AND DISCUSSIONS

### 5.1 Waiting Time by Simulation

#### 5.1.1 Headway distribution

When the vehicles arrive in random, the headway distribution is the negative exponential distribution. When the traffic volume is high, the movement of one vehicle affects or is affected by other vehicles. The Erlang distribution can explain the traffic condition in the intermediate state, which lies between the random and constant headway states (May, 1990). In this research, the Erlang distribution was assumed for conflicting headway. The K shape factor was varied to achieve the best fit. The headway distribution from the field data is shown in Figure 3. From the analysis, the Erlang-2 distribution (shape factor of 2) resulted in the least deviation.

#### 5.1.2 Critical headway

The collected data showed 171 drivers with inconsistent behavior, i.e. the largest rejected headway larger than the accepted headway. Other 3 drivers accepted headway equal to their largest rejected headway. Those drivers were excluded from the analysis. The remaining 649 pairs of largest rejected headway and accepted headway were used for the critical headway estimation by maximum likelihood method. The maximization of the log-likelihood function could be determined by the Microsoft Excel's Solver. Based on the calculation by Eq. (2) and

Eq. (3), the u-turn vehicle has the mean critical headway of 4.4 seconds and the standard deviation of 1.2 seconds. This implied that when a u-turn vehicle faced a headway of at least 4.4 seconds, the vehicle would probably make a u-turn. Since all drivers may not conduct the same behavior, this research considered the randomness of the critical headway in order to better represent the real world situation.

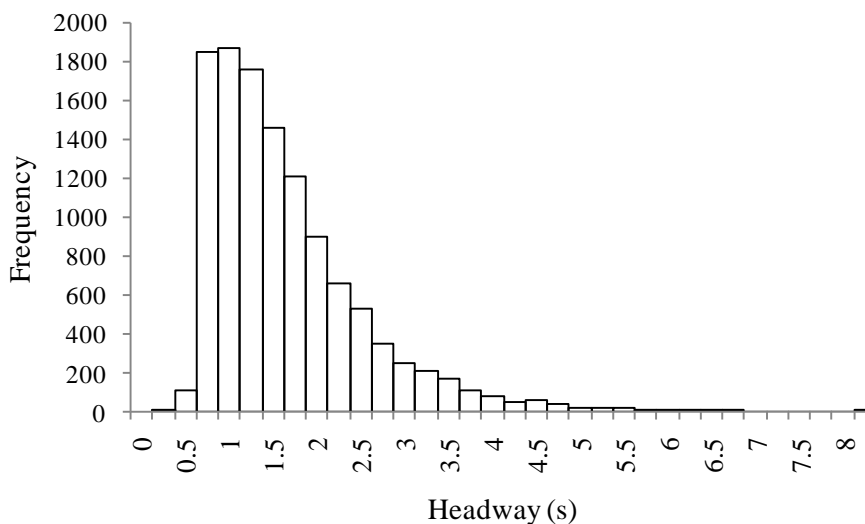


Figure 3. Histogram showing headway distribution

### 5.1.3 Estimated waiting time

From the above headway distribution information and critical headway characteristic, the spreadsheet simulation could be conducted to estimation the waiting time. The results of the simulation are summarized in Table 1. The larger amount of conflicting traffic volume makes the u-turn vehicles to wait longer.

The relationships between the waiting time and conflicting traffic volume were in exponential form. This is in accordance with the gap acceptance theory and past research (Madanat *et al.*, 1994; Chandra *et al.*, 2009). When the conflicting volume is larger, there is smaller chance of entering the traffic stream. The u-turn vehicle has to wait, in the increasing rate, for the acceptable headway.

From the simulation results, the waiting time model as a function of conflicting traffic volume can be developed as shown in Eq. (6). This model is based on the assumptions that the conflicting headway follows the Erlang-2 distribution and the u-turn critical headway equals 4.4 seconds with standard deviation of 1.2 seconds.

$$t_s = 2.169e^{1.189q_c/1000} \tag{6}$$

where,

- $t_s$  : u-turn waiting time (second), and
- $q_c$  : conflicting through traffic volume (vehicle/hour),

The simulation approach considers the random headway size according to the statistic distribution. The conflicting traffic includes all vehicles that oppose the movement of the u-turn vehicle no matter how many lanes on the roadway. When the conflicting flow rate is small, the generated headways are large. The conflicting vehicles can distribute on one lane or more. However, when the conflicting flow rate is large, the generated headways are small.

The conflicting vehicles cannot be on the same lane; more traffic lanes on roadway are required to accommodate such large traffic volume. Therefore, the estimated model can be applied when the conflicting traffic is in one or two lanes, given that the conflicting flow rate is not larger than the roadway capacity and the road configuration can accommodate u-turn maneuver.

Table 1. The results of estimated waiting time

Conflicting volume (vph)	Mean waiting time (second)	Standard deviation	Number of seeds	Error of prediction	Percent error
200	3.5	0.43	25	0.17	4.77%
400	3.8	0.15	15	0.07	1.94%
600	4.6	0.14	15	0.07	1.57%
800	5.7	0.25	15	0.13	2.24%
1000	7.2	0.37	15	0.19	2.63%
1200	9.1	0.47	15	0.24	2.64%
1400	11.5	0.67	15	0.34	2.94%
1600	14.6	1.03	15	0.52	3.58%
1800	18.5	1.24	15	0.63	3.38%
2000	23.5	1.59	15	0.80	3.42%
2200	29.3	2.25	20	0.99	3.37%
2400	37.4	2.69	30	0.96	2.58%
2600	47.5	3.36	50	0.93	1.96%
2800	60.2	5.00	100	0.98	1.63%
3000	77.1	7.47	220	0.99	1.28%
3200	97.4	11.11	475	1.00	1.02%

## 5.2 Waiting Time by Data Collection

From all periods of data collection, the conflicting flow rate and the waiting time of each u-turn vehicle were extracted in the laboratory. The data points were grouped into intervals of conflicting flow rate of 250 vehicles per hour. In each interval, there were some outliers and extreme data points. This might represent the unusual traffic situations of u-turn maneuver. Two main situations from the observation that created outlier data points include:

- the driver hesitated and missed a large headway, then had to wait for a long time to get another acceptable headway, and
- all the conflicting vehicles came in a line on the same lane (close to median) during some periods of time, resulting in the small size of headways. This situation is not usual for traffic operation on multilane roadway.

After screening those outliers and extreme data points, there were 768 u-turn vehicles remaining for the waiting time determination. The waiting time was averaged in each group of conflicting flow rate. The descriptive statistics of the waiting time is summarized in Table 2. Due to variability of driver behaviors in the real world, the observed waiting time had greater standard deviation than the simulated one. In addition, the observed data was grouped into intervals while the simulations were conducted according to exact conflicting flow rates. Like



the above results from simulation, the larger conflicting flow rate, the longer waiting time. The distribution of the amount of data points in each interval seemed to follow the normal distribution. The maximum number of data belonged to interval of 1,750-2,000 vehicles per hour. It should be noted that the first and the last intervals had only a few data points.

Table 2. The results of observed waiting time

Conflicting volume (vph)	Number of data points	Mean waiting time (second)	Standard deviation	Standard error
250-500	2	5.61	3.29	2.32
500-750	14	5.34	2.46	0.66
750-1000	51	7.27	3.55	0.50
1000-1250	63	10.92	5.22	0.66
1250-1500	113	13.94	8.62	0.81
1500-1750	126	19.13	10.39	0.93
1750-2000	152	21.95	14.08	1.14
2000-2250	119	28.07	16.15	1.48
2250-2500	66	30.12	17.76	2.19
2500-2750	41	43.61	30.70	4.79
2750-3000	18	50.34	25.14	5.93
3000-3250	3	70.74	30.82	17.79
Total	768			

The conflicting traffic volume is in the unit of vehicle per hour (vph). The headway acceptance process considers all conflicting vehicles that oppose the u-turn vehicle movement regardless of traffic lane. In order to determine the more general result in the unit of vehicle per hour per lane (vphpl), the lane distribution should be studied in future. A past study in Florida showed that the conflicting traffic volume for u-turns on six-lane streets equals 2.2 times the average opposing major street traffic volume in each lane (Liu *et al.*, 2009).

### 5.3 Waiting Time Comparison

In this section, the waiting time estimated from the simulation is compared with the observed waiting time from field data collection. Figure 4 illustrates such a comparison. The line shows the estimated waiting time from simulation while the dots represent the observed waiting time from data collection, using midpoint of conflicting flow rate intervals.

The observed waiting time is close to the estimated waiting time when the through traffic volume is not large (less than 2,200 vph). When the through traffic volume is large (more than 2,200 vph), the observed waiting time is lower than the estimated value. The waiting time estimation is based on the conflicting traffic volume, headway distribution, u-turn mean critical headway and standard deviation. Those factors are in an aggregate manner. The model does not capture any specific driver behaviors, which might vary according to the amount of conflicting traffic volume. When the conflicting flow rate is low, there is a high possibility to find large headways. The drivers feel relax and do not mind missing the first acceptable headway. They are willing to wait for another acceptable headway, which do not take a long time. So, the waiting time from the field is a bit longer than the estimated waiting time. On the contrary, when the drivers face a lot of conflicting vehicles,

they tend to make u-turn as soon as possible to avoid a long waiting time. They do not want to miss the first available large headway. In this situation, the u-turn drivers may select smaller headway for u-turn. Therefore, the waiting time from the field is lower than the estimated value. This forcing u-turn behavior creates unsafe and unstable traffic conditions. Some accidents may happen, e.g. collision between u-turn vehicle and through vehicle or collision among through vehicles who cannot stop their cars safely. The through traffic stream may be interrupted and create traffic shock wave. The comparison implied that the u-turn operation at this site might not be safe and require traffic control when the conflicting traffic flow was larger than 2,200 vph.

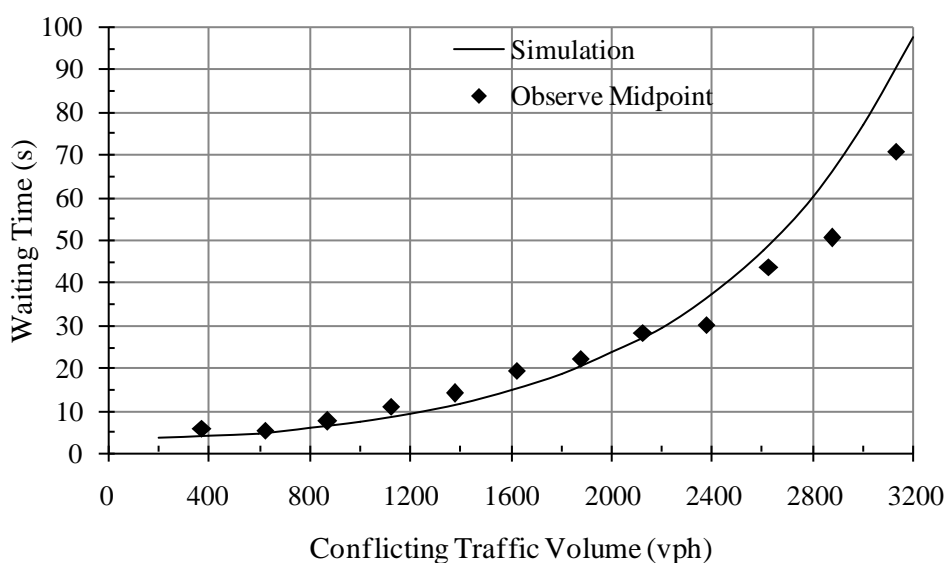


Figure 4. Waiting time comparison: simulation versus field data

In order to improve the model estimation, one possible approach is to separate into two models, one for small conflicting volume and the other for large conflicting volume. The traffic characteristics for both conditions may be different. This requires two sets of data collection and analysis. Another possible approach is to add a term in the model in order to capture effects of driver behaviors. The term should relate to conflicting traffic volume. Those improvements will be considered in the future studies.

## 6. CONCLUSIONS

This paper proposed the spreadsheet simulation method to estimate the waiting time of the u-turn vehicle, based on the stochastic random headway generation according to the known probability distributions. The inputs of the estimation process included the conflicting headway distribution type and the u-turn mean critical headway and standard deviation. It should be noted that the proposed simulation method relied on some assumptions as stated above in the methodology section. The results showed that the waiting time increases as the conflicting flow rate increases. The relationship was in the exponential form. This paper only considered the waiting time of leading vehicles of u-turn traffic because those vehicles could decide to make unsafe forcing u-turns. However, for a higher u-turn traffic flow, the follow-up u-turns are common and will be considered in the future studies. The queuing time may also be analyzed.

This research also collected the field data in order to determine the inputs for the simulation and investigate the difference between the theory and the real world. The analysis results concluded that the estimated waiting time can reasonably represent the field waiting time when the conflicting flow rate is less than 2,200 vehicles per hour. When the conflicting flow rate is larger, the simulated waiting time are overestimated. The observed waiting time is much lower than its estimated value. This implies that the u-turn drivers became impatient while waiting for a long time and conducted the unsafe forcing u-turn maneuver.

The waiting time characteristics of u-turn vehicle are important because the driver may take a risk after waiting for a long time. In practice, there are some kinds of control at the u-turn junction to avoid unsafe operation and facilitate the traffic flow. In most major u-turn junctions in Bangkok, the traffic police control the traffic movement during peak periods. The results in this paper can be used as a guideline for controlling the u-turn movement.

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